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Transient solution of a two homogeneous servers finite capacity Markovian Queueing system with Environmental and Catastrophic effects

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Abstract

In the present paper we consider a two homogeneous servers Markovian queueing system with environmental and catastrophic effects. Time dependent solution is obtained by using probability generating function technique and further the steady state probabilities of system size are also derived. Some measure of effectiveness and particular cases of the model have also been derived and discussed.

Keywords: Markovian queueing system, catastrophes, environmental effects and Laplace transforms.

1. Introduction

The simple Markovian birth and death queueing models have been the object of systematic investigation for a long time. In recent years the attention has been focused on the effect of catastrophes. When a catastrophes occurs, all present customers are flushed out and lost and the servers are ready for servie when new arrival occurs. There is a considerable litrature on queues with catastrophes (see e.g. [2,3,4,6,7,8]). The catastrophized processes with environmental effects may be suitable to apporoach in Biological sciences (see e. g. [1,5]). We found that the change in the environment affects the state of the queueing system. In other words, the state of the queueing system is a function of environmental change factors. The direct application of the model can be described to a biological phenomenon that there are many creatures such as cochroaches, ants etc whose movement is restricted when we put up a sepray on them (catastrophes) and also with the change of temperature(environment). As the temperature drops below a critical temperature say T_0 , the movement (production) of such like creatures becomes

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almost zero . On the other hand , as the temperature goes higher than T_0 the movement becomes normal. In the current study we extend the analysis to deal simulteneously with system that suffer random catastrophes with environmental effects and two identical servers. It is not necessary that a queueing system will have only one servers. Practically they may have more than one servers identical or non identical in their functioning. Here in this paper we consider the case of two servers with identical rate in each of the environmental state. The time dependent and steady state solution have been obtained by using probability generating function techniques.Further some particular cases are also derived and discussed.

2. Assumptions and Definitions

- The customers arrive in the system one by one in accordance with a possion process at a single service station. The arrival pattern is non-homogeneous, i.e.there may exist two arrival rates, namely λ_1 and 0 of which only one is operative at any instant.

- Here we consider two servers and the service time distribution is exponentially distributed. further, corresponding to arrival rate λ_1 the possion service rate is μ_1 for both the servers and the service rate corresponding to the arrival rate 0 is μ_2 for both the servers.

The state of the system when operating with arrival rate λ_1 and service rate μ_1 is designated as E whereas the other with arrival rate 0 and service rate μ_2 is designated as F.

- The Possion rates at which the system moves from environmental states F to E and E to F are denoted by α and β respectively.

- The catastrophe occur according to a poisson process with rate ζ . The effect of each catastrophe is to make the queue instantly empty. Simultaneously, the system becomes ready to accept the new customers.

- The queue discipline is first-come-first-served.

- The capacity of the system is limited to M i.e.,if at any instant there are M units in the queue then the units arriving at that instant will not be permitted to join the queue, it well be considered lost for the system.

Define,

$P_n(t)$=Joint probability that at time t the system is in state E and n units are in the queue,including the one in service.

$Q_n(t)$=Joint probability that at time t the system is in state F and n units are in the queue,including the one in service.

$R_n(t)$=The probability that at time t there are n units in the queue, including the one in service.

$$R_n(t) = P_n(t) + Q_n(t) \tag{1}$$

Let us reckon time t from an instant when there are zero customers in the queue and the system is in the environmental state E so that the initial conditions associated with $P_n(t)$ and $Q_n(t)$ becomes,

$$P_n(0) = \begin{cases} 1; & n = 0 \\ 0; & otherwise \end{cases}$$

$$Q_n(0) = 0; \quad \forall n \tag{2}$$

3. Equations Governing the System

$$\frac{d}{dt}P_0(t) = -(\lambda_1 + \beta + \zeta)P_0(t) + \mu_1 P_1(t) + \alpha Q_0(t) + \zeta \sum_{n=0}^{M} P_n(t); \quad n = 0 \tag{3}$$

$$\frac{d}{dt}P_1(t) = -(\lambda_1 + \mu_1 + \beta + \zeta)P_1(t) + 2\mu_1 P_2(t) + \alpha Q_1(t) + \lambda_1 P_0(t); n = 1 \tag{4}$$

$$\frac{d}{dt}P_n(t) = -(\lambda_1 + 2\mu_1 + \beta + \zeta)P_n(t) + 2\mu_1 P_{n+1}(t) + \lambda_1 P_{n-1}(t) + \alpha Q_n(t) \tag{5}$$

$$\frac{d}{dt}P_M(t) = -(2\mu_1 + \beta + \zeta)P_M(t) + \lambda_1 P_{M-1}(t) + \alpha Q_M(t); n = M \tag{6}$$

$$\frac{d}{dt}Q_0(t) = -(\alpha + \zeta)Q_0(t) + \mu_2 Q_1(t) + \beta P_0(t) + \zeta \sum_{n=o}^{M} Q_n(t); n = 0 \tag{7}$$

$$\frac{d}{dt}Q_1(t) = -(\mu_2 + \alpha + \zeta)Q_1(t) + 2\mu_2 Q_2(t) + \beta P_1(t) \tag{8}$$

$$\frac{d}{dt}Q_n(t) = -(2\mu_2 + \alpha + \zeta)Q_n(t) + 2\mu_2 Q_{n+1}(t) + \beta P_n(t); 0 < n < M \tag{9}$$

$$\frac{d}{dt}Q_M(t) = -(2\mu_2 + \alpha + \zeta)Q_M(t) + \beta P_M(t); n = M \tag{10}$$

4. Transient Analysis

Let, the Laplace Transform of $f(t)$ be

$$\bar{f}(s) = \int_0^\infty e^{-st} f(t)dt \tag{11}$$

Taking Laplace tranform of the equations (3)-(10) and using the initial conditions (2), we get,

$$(s + \lambda_1 + \beta + \zeta)\overline{P_0}(s) - 1 = \mu_1 \overline{P_1}(s) + \alpha \overline{Q_0}(s) + \zeta \sum_0^M \overline{P_n}(s); n = 0 \tag{12}$$

$$(s + \lambda_1 + \mu_1 + \beta + \zeta)\overline{P_1}(s) = 2\mu_1 \overline{P_2}(s) + \alpha \overline{Q_1}(s) + \lambda_1 \overline{P_0}(s); n = 1 \tag{13}$$

$$(s + \lambda_1 + 2\mu_1 + \beta + \zeta)\overline{P_n}(s) = 2\mu_1 \overline{P_{n+1}}(s) + \lambda_1 \overline{P_{n-1}}(s) + \alpha \overline{Q_n}(s); \tag{14}$$

$$(s + 2\mu_1 + \beta + \zeta)\overline{P_M}(s) = \lambda_1 \overline{P_{M-1}}(s) + \alpha \overline{Q_M}(s) \tag{15}$$

$$(s + \alpha + \zeta)\overline{Q_0}(s) = \mu_2 \overline{Q_1}(s) + \beta \overline{P_0}(s) + \zeta \sum_{n=0}^M \overline{Q_n}(s) \tag{16}$$

$$(s + \alpha + \mu_2 + \zeta)\overline{Q_1}(s) = 2\mu_2 \overline{Q_2}(s) + \beta \overline{P_1}(s) \tag{17}$$

$$(s + 2\mu_2 + \alpha + \zeta)\overline{Q_n}(s) = 2\mu_2 \overline{Q_{n+1}}(s) + \beta \overline{P_n}(s); \tag{18}$$

$$(s + 2\mu_2 + \alpha + \zeta)\overline{Q_M}(s) = \beta \overline{P_M}(s); \tag{19}$$

Define, The probability generating function by ,

$$P(z,s) = \sum_{n=0}^M \overline{P_n}(s) z^n \tag{20}$$

$$Q(z,s)=\sum_{n=0}^{M}\overline{Q}_n(s)z^n \tag{21}$$

$$R(z,s)=\sum_{n=0}^{M}\overline{R}_n(s)z^n \tag{22}$$

where

$$R(z,s)=P(z,s)+Q(z,s) \tag{23}$$

and

$$\overline{R}_n(s)=\overline{P}_n(s)+\overline{Q}_n(s) \tag{24}$$

Multiplying equations (12)-(15) by z^n, Summing over the respective region of n and using equations (20)-(22),we have

$$(s+\lambda_1+\beta+\zeta+2\mu_1)\sum_{n=0}^{M}\overline{P}_n(s)z^n-\frac{2\mu_1}{z}[\sum_{n=0}^{M-1}\overline{P}_{n+1}(s)z^{n+1}-\overline{P}_0(s)+\overline{P}_0(s)-\overline{P}_1(s)$$

$$+\overline{P}_1(s)]-\lambda_1 z[\sum_{n=1}^{M}\overline{P}_{n-1}(s)z^{n-1}+z^M\overline{P}_M(s)-z^M\overline{P}_M(s)]-\alpha\sum_{n=0}^{M}\overline{Q}_n(s)z^n$$

$$-2\mu_1\overline{P}_0(s)-\lambda_1\overline{P}_M(s)z^M-1-\zeta\sum_{n=0}^{M}\overline{P}_n(s)-\mu_1 z\overline{P}_1(s)-\mu_1\overline{P}_1(s)=0$$

$$[(s+\lambda_1+\beta+\zeta+2\mu_1)z-\lambda_1 z^2-2\mu_1]P(z,s)-\alpha zQ(z,s)+2\mu_1(1$$

$$-z)\overline{P}_0(s)-\lambda_1(1-z)\overline{P}_M(s)z^{M+1}-z-z\zeta\sum_{n=0}^{M}\overline{P}_n(s)+z\mu_1(1$$

$$-z)\overline{P}_1(s)=0 \tag{25}$$

Similarly, from equations (16)-(19) on using equations (20)-(22),we have

$$(s+2\mu_2+\alpha+\zeta)\sum_{n=0}^{M}\overline{Q}_n(s)z^n-\frac{2\mu_2}{z}[\sum_{n=0}^{M-1}\overline{Q}_n(s)-\overline{Q}_0(s)+\overline{Q}_0(s)+\overline{Q}_1(s)$$

$$-\overline{Q}_1(s)]-\beta\sum_{n=0}^{M}\overline{P}_n(s)z^n-\zeta\sum_{n=0}^{M}\overline{Q}_n(s)-2\mu_2\overline{Q}_0(s)-\mu_2\overline{Q}_1(s)-\mu_2 z\overline{Q}_1(s)=0$$

$$[2\mu_2-z(s+2\mu_2+\alpha+\zeta)]Q(z,s)+\beta zP(z,s)+\zeta z\sum_{n=0}^{M}\overline{Q}_n(s)$$

$$-2\mu_2\overline{Q}_0(s)(1-z)-z\mu_2(1-z)\overline{Q}_1(s)=0 \tag{26}$$

From eq.(25)

$$P(z,s)=\alpha zQ(z,s)-2\mu_1(1-z)\overline{P}_0(s)+\lambda_1(1-z)\overline{P}_M(s)z^{M+1}+z+\zeta z\sum_{n=0}^{M}\overline{P}_n(s)$$

$$-\mu_1 z\overline{P}_1(s)(1-z)/[z(s+\lambda_1+2\mu_1+\beta+\zeta)-\lambda_1 z^2-2\mu_1]$$

From eq.(26),

$$Q(z,s)=\frac{2\mu_2(1-z)\overline{Q}_0(s)-\beta zP(z,s)-\zeta z\sum_{n=0}^{M}\overline{Q}_n(s)+z\mu_2(1-z)\overline{Q}_1(s)}{[2\mu_2-z(s+2\mu_2+\alpha+\zeta)]}$$

Putting the value of $Q(z,s)$ in (25)

$$P(z,s)=2\alpha z\mu_2(1-z)\overline{Q}_0(s)-\alpha\zeta z^2\sum_{n=0}^{M}\overline{Q}_n(s)-2\mu_1(1-z)\overline{P}_0(s)[2\mu_2$$

$$-z(s+2\mu_2+\alpha+\zeta)]+\lambda_1(1-z)\overline{P}_M(s)z^{M+1}[2\mu_2-z(s+2\mu_2$$

$$+\alpha+\zeta)]+z[2\mu_2-z(s+2\mu_2+\alpha+\zeta)]+\zeta z\sum_{n=0}^{M}\overline{P}_n(s)[2\mu_2$$

$$-z(s+2\mu_2+\alpha+\zeta)]-\mu_1(1-z)z\overline{P}_1(s)[2\mu_2$$

$$-z(s+2\mu_2+\alpha+\zeta)]+\mu_2(1-z)z^2\alpha\overline{Q}_1(s)/[z(s+\lambda_1+2\mu_1$$

$$+\beta+\zeta)-\lambda_1 z^2-2\mu_1][2\mu_2-z(s+2\mu_2+\alpha+\zeta)+\beta z^2\alpha] \tag{27}$$

Similarly, putting the value of $P(z,s)$ in eq.(26), we get

$$Q(z,s) = 2\mu_1(1-z)\beta z\overline{P}_0(s) + 2\mu_2(1-z)[(s+\lambda_1+\beta+\zeta+2\mu_1)z$$

$$-\lambda_1 z^2 - 2\mu_1]\overline{Q}_0(s) - \beta\lambda_1(1-z)\overline{P}_M(s)z^{M+2} - \beta z^2$$

$$-\beta z^2\zeta\sum_{n=0}^{M}\overline{P}_n(s) + \zeta z[\lambda_1 z^2 + 2\mu_1 - (s+\lambda_1+\beta+\zeta$$

$$+2\mu_1)z]\sum_{n=0}^{M}\overline{Q}_n(s) + \mu_1 z^2(1-z)\beta\overline{P}_1(s) + z\mu_2(1-z)[(s+\lambda_1$$

$$+\beta+\zeta+2\mu_1)z - \lambda_1 z^2 - 2\mu_1]\overline{Q}_1(s)/\beta\alpha z^2 + [2\mu_2 - z(s$$

$$+2\mu_2\alpha+\zeta)][(s+\lambda_1+\beta+\zeta+2\mu_1)z - \lambda_1 z^2 - 2\mu_1] \tag{28}$$

Now from equation(23), we have

$$R(z,s) = [\alpha z + \mu_2(s+\lambda_1+\beta+\zeta+\mu_1)z - \lambda_1 z^2 - \mu_1]2\mu_2(1-z)\overline{Q}_0(s)$$

$$+\zeta z\sum_{n=0}^{M}\overline{Q}_n(s)[\lambda_1 z^2 - z(s+\lambda_1+\mu_1+\beta+\zeta) + 2\mu_1 - \alpha z] + (1$$

$$-z)\overline{P}_0(s)2\mu_1[z(s+2\mu_2+\alpha+\zeta) - 2\mu_2 + z\beta] + (1-z)\overline{P}_M(s)\lambda_1[z^{M+1}2\mu_2$$

$$-z(s+2\mu_2+\alpha+\zeta) - \beta z^{M+2}] + \zeta z\sum_{n=0}^{M}\overline{P}_n(s)[2\mu_2 - z(s+2\mu_2+\alpha+\zeta)$$

$$-\beta z] + z[2\mu_2 - z(s+2\mu_2+\alpha+\zeta)] - \beta z^2 + \overline{P}_1(s)z\mu_1(1-z)[z\beta - [2\mu_2$$

$$-z(s+2\mu_2+\alpha+\zeta)]] + z\mu_2(1-z)\overline{Q}_n(s)[z\alpha + z(s+2\mu_1+\lambda_1+\zeta+\beta)$$

$$-2\mu_1 - \lambda_1 z^2]/-z^2 s^2 + s[\lambda_1 z^3 - z^2(\lambda_1+2\mu_1+2\mu_2+\alpha+\beta+2\zeta)$$

$$+z(2\mu_1+2\mu_2)] - z^2\zeta(\beta+\alpha+\zeta) + (1-z)[-z^2\lambda_1(2\mu_2+\alpha+\zeta)$$

$$+z[2\alpha\mu_1 + 2\mu_2(2\mu_1+\lambda_1+\beta+\zeta)] - 4\mu_2\mu_1] \tag{29}$$

The unknown quantities in equcation (29) are determined as follows:

Setting z=1, in equations (27) and (28) respectively,we have

$$P(1,s)=\sum_{n=0}^{M}\overline{P}_n(s)=\frac{(s+\alpha+\zeta)}{s(s+\beta+\alpha+\zeta)} \tag{30}$$

$$Q(1,s)=\sum_{n=0}^{M}\overline{Q}_n(s)=\frac{\beta}{s(s+\beta+\alpha+\zeta)} \tag{31}$$

Further, relation (29) is a polynomial in z and exists for all values of z, including the three zeros of the denominator. Hence $\overline{P}_0(s)$, $\overline{Q}_0(s)$, $\overline{R}_M(s)$ are obtained by setting the numerator equal to zero and substituting the three zeros a_1, a_2, a_3 (say) of the denominator (at each of which the numerator must vanish).

The Laplace transform of various state probabilities for the number of units in the queue, including the one in service can be picked up as the co-efficient of the different powers of z in the expansion of equation(29).

5. Particular Case

Now letting $\alpha\to\infty$, $\beta\to\infty$ and setting $\mu_1=\mu_2=\mu$ (say)in relation (29), we have

$$r(z,s)=2\mu(1-z)\overline{R}_0(s)-\lambda_1 z^{M+1}(1-z)\overline{P}_M(s)+z(1-z)\mu\overline{R}_1(s)-\frac{\zeta z}{s}/\lambda_1 z^2$$

$$-z(s+\lambda_1+2\mu+\zeta)+2\mu \tag{32}$$

$$\overline{R}_0(s)=\overline{P}_0(s)+\overline{Q}_0(s)$$

$$r(z,s)=\lim_{\beta\to 0}[\lim_{\alpha\to\infty}R(z,s)]$$

Relation (32) is a polynomial in z and exists for all values of z,including the two zeros of the denominator.Hence R_0 and P_M can be obtained by setting the numerator equal to zero. Substituting the two zeros a_1 and a_2 (say)of the denominator (at each of which the numerator must vanish).Equating the denominating zero and we get Two roots a_1 and a_2.

6. Steady State Results

$$\lim_{t\to\infty} f(t) = \lim_{s\to 0} s\bar{f}(s) \text{ [By final value theorem]}$$

Thus if

$$R(z) = \sum_{n=0}^{M} \overline{R}_n z^n$$

where

$$R_n = \lim_{s\to 0} s\overline{R}_n(s)$$

Then

$$R(z) = \lim_{s\to 0} sR(z,s)$$

and

$$\sum_{n=0}^{M} R_n = \sum_{n=0}^{M} P_n + \sum_{n=0}^{M} Q_n = 1 \tag{33}$$

By using this property, we have from equation (29)

$$R(z) = 2\mu_2(1-z)[\alpha z + (\lambda_1 + \beta + \zeta + 2\mu_1)z - \lambda_1 z^2 - 2\mu_1]Q_0 + [\zeta z/(\alpha + \beta$$
$$+ \zeta)]\beta[\lambda_1 z^2 - z(\lambda_1 + 2\mu_1 + \beta + \zeta) - \alpha z + 2\mu_1] + (1-z)2\mu_1 P_0[z(2\mu_2$$
$$+ \alpha + \zeta) - 2\mu_2 + \beta z] + (1-z)\lambda_1 z^{M+1} P_M [2\mu_2 - z(2\mu_2 + \alpha + \zeta) - \beta z]$$
$$+ \zeta z(\alpha + \zeta)/(\alpha + \beta + \zeta)[2\mu_2 - z(2\mu_2 + \alpha + \zeta) - \beta z] + \mu_1 z(1-z)P_1[z\beta$$
$$-(2\mu_2 + \alpha + \zeta)z] + \mu_2 z(1-z)Q_1[z\alpha + (\lambda_1 + 2\mu_1 + \beta + \zeta)z - 2\mu_1$$
$$-\lambda_1 z^2]/z^3\lambda_1(\alpha + 2\mu_2 + \zeta) - z^2[\lambda_1(\alpha + 2\mu_2 + \zeta) + 2\alpha\mu_1 + 2\mu_2(\lambda_1 + 2\mu_1$$
$$+ \beta + \zeta) + \zeta(\alpha + \beta + \zeta)] + z[\{\alpha 2\mu_1 + mu_2(\lambda_1 + 2\mu_1 + \beta + \zeta) + 4\mu_1\mu_2\}$$
$$-4\mu_1\mu_2] \tag{34}$$

or,we can write,

$$R(z) = T(z)Q_0 + N(z)P_0 + L(z)P_M + M(z) + S(z)P_1 + R(z)Q_1/K(z) \qquad (35)$$

where $T(z), N(z)$ and $L(z)$, $S(z)$, $R(z)$ are the coeffient of Q_0, P_0, P_M, Q_1, and P_1 respectively in the numerator of equation (34) and $K(z)$ is the denominator of equation (34).

Equation (35) is a polynomial in z and exists for all values of z, including three zeros of the denominator. Hence Q_0, P_0, P_1 Q_1 and P_M can be obtained by setting the numerator equal to zero.Substituting the zeros b_1, b_2 and b_3 (say) the denominator (at each of which the numerator must vanish).

Now, three equations determining the constants Q_0, P_0, P_M, P_1 and Q_1 are

$$T(b_1)Q_0 + N(b_1)P_0 + L(b_1)P_M + S(b_1)P_1 + R(b_1)Q_{b_1} = -M(b_1) \qquad (36)$$

$$T(b_2)Q_0 + N(b_2)P_0 + L(b_2)P_M S(b_2)P_1 + R(b_2)Q_1 = -M(b_2 \qquad (37)$$

$$T(b_3)Q_0 + N(b_3)P_0 + L(b_3)P_M S(z)P_1 + R(z)Q_1 S(b_3)P_1 + R(b_3)Q_1 = -M(b_3) \qquad (38)$$

After solving these equation, we have

$$Q_0 = \frac{-A_{11}M(b_1) + A_{21}M(b_2) - A_{31}M(b_3)}{A}$$

$$P_0 = \frac{A_{12}M(b_1) - A_{22}M(b_2) + A_{32}M(b_3)}{A}$$

$$P_M = \frac{A_{13}M(b_1) + A_{23}M(b_2) - A_{33}M(b_3)}{A}$$

$$P_1 = \frac{A_{14}M(b_1) - A_{24}M(b_2) + A_{34}M(b_3)}{A}$$

$$Q_1 = \frac{-A_{15}M(b_1) + A_{25}M(b_2) - A_{35}M(b_3)}{A}$$

where,

$$A = \left\| \begin{matrix} T(b_1) & N(b_1 & L(b_1) & S(b_1 & R(b_1)) \\ T(b_2) & N(b_2 & L(b_2) & S(b_2 & R(b_2) \\ T(b_3) & N(b_3 & L(b_3) & S(b_3 & R(b_3) \\ & & & & \\ & & & & \end{matrix} \right\|$$

A_{ij} is the co-factor of the $(i, j)^{th}$ element of A.

By putting the values of Q_0, P_0, P_1, Q_1 and P_M in equation (35), we have

$$R(z) = T(z)[-M(b_1)A_{11} + M(b_2)A_{21} - M(b_3)A_{31}] + N(z)[M(b_1)A_{12}$$

$$-M(b_2)A_{22} + M(b_3)A_{32}] + L(z)[-M(b_1)A_{13} + M(b_2)A_{23} - M(b_3)A_{33}]$$

$$+S(z)[-M(b_1)A_{14} - M(b_2)A_{24} + M(b_3)A_{34}] + R(z)[-M(b_1)A_{15} + M(b_2)A_{25}$$

$$-M(b_3)A_{35}] + A.M(z)/A.K(z) \tag{39}$$

7. Mean Queue Length

Define,

L_q =Expected number of customers in the queue excluding the one in service.

then

$$L_q = [R'(z)]_{z=1}$$

$$R'(z) = AK(z)[T'(z)[-M(b_1)A_{11} + M(b_2)A_{21} - M(b_3)A_{31}] + N'(z)[M(b_1)A_{12}$$

$$-M(b_2)A_{22} + M(b_3)A_{32}] + L'(z)[-M(b_1)A_{13} + M(b_2)A_{23} - M(b_3)A_{33}]$$

$$+R'(z)[-M(b_1)A_{15} + M(b_2)A_{25} - M(b_3)A_{35}] + S'(z)[M(b_1)A_{14} - M(b_2)A_{24}$$

$$+M(b_3)A_{34}] + AM'(z)] - (T(z)[-M(b_1)A_{11} + M(b_2)A_{21} - M(b_3)A_{31}] + L(z)[$$

$$-M(b_1)A_{13}+M(b_2)A_{23}-M(b_3)A_{33}]+N(z)[M(b_1)A_{12}-M(b_2)A_{22}$$

$$+M(b_3)A_{32}]+S(z)[M(b_1)A_{14}-M(b_2)A_{24}+M(b_3)A_{34}]+R(z)[-M(b_1)A_{15}$$

$$+M(b_2)A_{25}-M(b_3)A_{35}]+AM(z))AK'(z)/A^2[K(z)]^2$$

Therefore from equcation (39) we have,

$$L_q = K(1)(T'(1)[-M(b_1)A_{11}+M(b_2)A_{21}-M(b_3)A_{31}]+N'(1)[M(b_1)A_{12}$$

$$-M(b_2)A_{22}+M(b_3)A_{32}]+L'(1)[-M(b_1)A_{13}+M(b_2)A_{23}-M(b_3)A_{33}]$$

$$+R'(1)[-M(b_1)A_{15}+M(b_2)A_{25}-M(b_3)A_{35}]+S'(1)[M(b_1)A_{14}-M(b_2)A_{24}$$

$$+M(b_3)A_{34}+AM'(1)])-(T(1)[-M(b_1)A_{11}+M(b_2)A_{21}-M(b_3)A_{31}]+L(1)[$$

$$-M(b_1)A_{13}+M(b_2)A_{23}-M(b_3)A_{33}]+N(1)[M(b_1)A_{12}-M(b_2)A_{22}$$

$$+M(b_3)A_{32}]+S(1)[M(b_1)A_{14}-M(b_2)A_{24}+M(b_3)A_{34}]+R(1)[-M(b_1)A_{15}$$

$$+M(b_2)A_{25}-M(b_3)A_{35}]+AM(1))K'(1)/A[K(1)]^2 \tag{40}$$

where dashes denotes the first derivative with respect to z.

8. Particular Case

Relation (32), on applying the theory of Laplace transform gives

$$r(z,s)=\frac{2\mu(1-z)\overline{R}_0(s)-\lambda_1 z^{M+1}(1-z)\overline{P}_M(s)-z-\dfrac{\zeta z}{s}+z(1-z)\mu\overline{R}_1(s)}{\lambda_1 z^2-z(s+\lambda_1+2\mu+\zeta)+2\mu}$$

$$r(z)=\frac{2\mu(1-z)R_0-\lambda_1 z^{M+1}(1-z)P_M-\zeta z+z(1-z)\mu R_1}{\lambda_1 z^2-z(\lambda_1+2\mu+\zeta)+2\mu} \tag{41}$$

where

$$r(z)=\lim_{s\to 0} sr(z,s)$$

Equation (41) is a polynomial in z and exists for all values of z, including the two zeros of the denominator.Hence we can obtained the term R_0 and P_M by setting the numerator equal to zero. Substituting the two zeros a_1 and a_2 (say)of the denominator (at each of which the numerator must vanish).Equating the denominating zero and we get Two roots a_1 and a_2.

Two equation determining the constants R_0 and P_M are,

$$2\mu(1-a_1)R_0 - \lambda_1 a_1^{M+1}(1-a_1)P_M + a_1(1-a_1)\mu R_1 = \zeta a_1 \tag{42}$$

$$2\mu(1-a_2)R_0 - \lambda_1 a_2^{M+1}(1-a_2)P_M + a_2(1-a_2)\mu R_1 = \zeta a_2 \tag{43}$$

On solving these equations, we have

$$R_1 = k(constant)$$

$$P_M = \frac{(a_1 - a_2)[\lambda_1 + \mu k]}{\lambda_1[a_1^{M+1} - a_2^{M+1}]}$$

and

$$R_0 = \frac{\zeta a_1}{(1-a_1)2\mu} + \frac{(\lambda_1 + \mu k)}{2\mu} a_1^{M+1} \frac{a_1 - a_2}{a_1^{M+1} - a_2^{M+1}} - \frac{a_1 \mu k}{2\mu}$$

where

$$\lambda_1[(1-a_2)+(1-a_2)] = -\zeta, \quad 1cm a_1 > 1$$

Putting the value of R_0, P_M and R_1 in equation (41) and we get a well known result of the $M/M/2$ queue with finite waiting space M

9. Concluding Remarks

In the present paper we consider a two homogeneous servers Markovian queueing system with environmental and catastrophic effects. The direct application of the model can be described to a biological phenomenon that there are many creatures such as cochroaches, ants etc whose movement is restricted when we put up a sepray on them (catastrophes) and also with the change of temperature(environment). As the temperature drops below a critical temperature say T_0 , the movement (production) of such like creatures becomes almost zero . On the other hand , as the temperature goes higher than T_0 the movement becomes normal.

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