A Note on Modeling Interactions Between Criteria in MCDA

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Abstract

The motivation behind this note is the computation of interactions between criteria within the framework of Multiple Criteria Decision Aiding. The focus is the use of a generalization of the Choquet integral also known as the bi-capacity model. This model takes into consideration bipolar scales. The note introduces the Choquet integral in the bipolar scale. An application example consists of a comparative analysis of the uses of the bipolar Choquet integral and the ELECTRE IV method in a suppliers ranking problem. This problem has six alternatives and six evaluation criteria. The rankings of alternatives produced by the bipolar Choquet integral and the use of the ELECTRE IV method are then compared. It is concluded that there are advantages from using the bipolar Choquet integral for the multicriteria ranking of alternatives. The note closes with the suggestion that more extensive comparisons should be carried out.

Keywords: decision support systems, multicriteria decision analysis.

1. Introduction

One important problem in Multiple Criteria Decision Aiding (MCDA) is the existence of interactions between criteria. When using a multiple attribute utility (or value) theoretical model there are requirements that must be met if, for example, a linear additive function is to be used [4, 5]. An important mathematical model that has been used for modeling interactions between criteria is the Choquet integral [3]. In decision theory, the Choquet integral is a way to measure the expected utility of an uncertain event [7].

The Choquet integral is indeed a generalization of the weighted arithmetic mean and has been extensively used since the last decade in MCDA in modeling interactions between criteria [8, 9, 10, 11. Marichal [15] also modeled interactions between criteria with the Choquet integral. A critical analysis of the use of the Choquet integral for modeling interactions between criteria was presented by Roy [17] though. This last author has pointed out that the generalization of the Choquet integral known as bipolar model (or model with bi-capacities) should be utilized in order to capture some particular aspects of interactions between criteria.

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In this note we show a comparison between the uses of the bipolar Choquet integral and the ELECTRE IV method. This is an outranking method that does not require knowledge of criteria weights [16]. The comparison is accomplished for a ranking problem under multicriteria. Next the comparison between the use of the Choquet integral in the bipolar scale is compared against the results obtained from using ELECTRE IV for a suppliers ranking problem under multiple criteria. Our approach is therefore different from that of [6], who extended the notion of concordance index that is not present in ELECTRE IV. These three authors have shown that the Choquet integral can be applied in order to build a transitive and complete pre-order relation of type existing in the ELECTRE methods. An extension of the concordance index of ELECTRE methods in order to deal with interactions between criteria was then proposed.

2. The Choquet integral in the bipolar scale

Given a finite set $J = \{1,2,...,n\}$ a fuzzy measure μ is a function of the form: $\mu : 2^J \to [0,1]$ such that $\mu(\phi) = 0$ and μ(J) = 1 (boundary conditions); μ(C) ≥ μ(D) if $D \subseteq C, \forall\, C, D \in J$ (monotonicity condition). Let P(J) be a set of pairs of subsets of J: $P(J) = \{(C,D), C,D \subseteq J, C \cap D = \phi\}$. A bi-capacity μ in J is a function $\mu : P(J) \to [0,1\} X[0,1]$ such that $\mu(C,\phi) = (c,0)$ and $\mu(\phi, D) = (0, D), c, d \in [0,1]$; $\mu(J,\phi) = (1,0)$ and $\mu(\phi, J) = (0,1)$ (boundary conditions); for each $(C,D),(E,F) \in P(J)$ such that $E \subseteq C, D \subseteq F$ we have $\mu(C,D) = (c,d)$ and $\mu(E,F) = (e,f)$, $c,d,e,f \in [0,1]$ with $c \geq e$ and $d \geq f$ (monotonicity condition). Here we use the following notation: $\mu^+(C,D) = c$, $\mu^-(C,D) = d$. A bi-capacity $\hat{\mu}$, on the set J, is a function $\hat{\mu} : P(J) \to [-1,1]$ such that $\hat{\mu}(\phi,\phi) = 0$; $\hat{\mu}(J,\phi) = 1$ and $\hat{\mu}(\phi, J) = -1$ (boundary conditions); if $E \subseteq C, D \subseteq F$, then $\hat{\mu}(C,D) \geq \hat{\mu}(E,F)$ (monotonicity condition). From each bi-polar capacity μ in J, we can obtain a bi-capacity $\hat{\mu}$ in J: $\hat{\mu}(C,D) = \mu^+(C,D) - \mu^-(C,D), \forall C,D \in P(J)$.

For each $x \in R^n$: $x^+ = \max\{x,0\}$ is the positive part of x, for each $x \in R$; $x^- = \max\{-x,0\}$ is the negative part of x, for each $x \in R$; $x^+ = (x_1^+, x_2^+, ... x_n^+)$ is the positive part of $x(x_1, x_2, ... x_n) \in R^n$; $x^- = (x_1^-, x_2^-, ... x_n^-)$ is the negative part of $x(x_1, x_2, ... x_n) \in R^n$. Given $x \in R^n$ we consider a permutation (.) of the elements of J, such that $|x_{(1)}| \leq |x_{(2)}| \leq ... \leq |x_{(j)}| \leq |x_{(n)}|$. For each element $j \in J$ we have two subsets $C(j) = \{i \in J : x_i \geq |x_{(j)}|\}$ and $D(j) = \{i \in J : -x_i \geq |x_{(j)}|\}$. Considering a bi-capacity μ in J and a vector $x \in R^n$ we can define its bipolar Choquet integral of the positive part as follows: $Ch^{+}(x,\mu)=\sum_{j\in J^{>}}\left(\left|x_{(j)}\right|-\left|x_{(j-1)}\right|\mu^{+}\left(C_{(j)},D_{(j)}\right)\right)$, where $J=\left\{j\in J/\left|x_{j}\right|>0\right\}$. In the same way we formulate the bipolar Choquet integral of the negative part as $Ch^{-}(x,\mu)=\sum_{j\in J^{>}}\left(\left|x_{(j)}\right|-\left|x_{(j-1)}\right|\mu^{-}\left(C_{(j)},D_{(j)}\right)\right)$. Therefore the bipolar Choquet integral is $Ch^{B}(x,\mu)=C^{+}(x,\mu)+Ch^{-}(x,\mu)$.

Other authors have used the bipolar Choquet integral for tackling different problems. For example, [12] extended the bipolar Choquet integral representation towards bipolar Cumulative Prospect Theory. In [12] proposed the bipolar Choquet integral for the case in which the underlying scale is bipolar and provided a characterization of bipolar fuzzy integrals. In [13] extended the PROMÉTHÉE methods to the case of interacting criteria on a bipolar scale. In [2] proposed to integrate the SMAA methodology [14] with the Choquet integral preference model.

3. Comparing the uses of the bipolar Choquet integral and ELECTRE IV

To compare the Choquet integral using bipolar scale against ELECTRE IV we make use of the decision matrix taken from the doctoral thesis of [1]. This last author analyzed the problem of ranking suppliers within the context of group decision making by applying ELECTRE IV. Table 1 displays the judgments by experts on the relative importance of each supplying firm with respect to every evaluation criterion. Those criteria are the following: C_1 = Cost, C_2 = Culture, C_3 = Design, C_4 = Quality, C_5 = Time, C_6 = Experience. The criteria order is: $C_1 > C_5 > C_6 > C_2 = C_3 = C_4$.

Table 1 - Decision matrix of Criteria versus Alternative supplying firms

Criteria ↓	Firm # 1	Firm # 2	Firm # 3	Firm # 4	Firm # 5	Firm # 6
C_1	0.1	0.15	0.15	0.03	-0.03	0.05
C_2	4	2	1	1	4	3
C_3	5	2	2	1	4	5
C_4	4	4	3	4	3	3
C_5	0.1	0.05	-0.05	0.15	0.2	0.1
C_6	8	20	15	6	10	5

In order to run a sensitive analysis over the computation of the bipolar Choquet integral we adopt two hypothesis: (A) hypothesis # 1 (H_1): $C_1 > C_5 > C_6 > C_2 = C_3 = C_4$ with the following interactions between criteria: $\mu_1 = 0.35$, $\mu_2 = 0.5\mu_1$, $\mu_2 = \mu_3 = \mu_4$,

$\mu_5 = 0.6\mu_1$, $\mu_5 = 0.84\mu_1$, given that $\sum_{i=1}^{6} \mu_i = 1$; (B) hypothesis # 2 (H_2): $C_6 > C_1 = C_5 > C_2 = C_3 = C_4$ with the following interactions between criteria: $\mu_6 = 0.32$, $\mu_1 = 0.56\mu_6$, $\mu_5 = \mu_1$, $\mu_2 = 0.6\mu_5$, $\mu_2 = \mu_3 = \mu_4$, given that $\sum_{i=1}^{6} \mu_i = 1$. Next, we show the steps for computing the bipolar Choquet integral.

Step 1 – Determination of fuzzy measures

In Table 2 we present the fuzzy measures for each hypothesis and for every criterion.

Table 2- Fuzzy measures for each hypothesis and for every criterion

Individual fuzzy measures	Fuzzy measures H_1/H_2	Criteria
μ_1	0.35/ 0.18	C_1
μ_2	0.09/ 0.11	C_2
μ_3	0.09/ 0.11	C_3
μ_4	0.09/ 0.11	C_4
μ_5	0.21/ 0.18	C_5
μ_6	0.18/ 0.32	C_6

Step 2 – Determination of the fuzzified decision matrix

The fuzzified decision matrix is obtained by multiplying each element of decision matrix by its fuzzy measure. In Table 3 we present the results for hypothesis H_1. An equivalent table has been determined for hypothesis H_2.

Table 3 - Fuzzified decision matrix for hypothesis H_1

Criteria ↓	Firm # 1	Firm # 2	Firm # 3	Firm # 4	Firm # 5	Firm # 6
C_1	0.035	0.0525	0.0525	0.0105	-0.0105	0.0175
C_2	0.3528	0.1764	0.0882	0.0882	0.3528	0.2646
C_3	0.441	0.1764	0.1764	0.0882	0.3528	0.441
C_4	0.3528	0.3528	0.2646	0.3528	0.2646	0.2646
C_5	0.021	0.0105	-0.0105	0.0315	0.042	0.021
C_6	1.4112	3.528	2.646	1.0584	1.764	0.882

Step 3: Computation of the bipolar Choquet integral

In Table 4 we display all the computational results for hypothesis H_1. Equivalent results are obtained for hypothesis H_2.

Table 4 - Computation of the bipolar Choquet integral

Criteria ↓	Firm # 1	Firm # 2	Firm # 3	Firm # 4	Firm # 5	Firm # 6
C_1	0.035	0.0525	0.0525	0.0105	-0.0105	0.0175
C_2	0.3528	0.1764	0.0882	0.0882	0.3528	0.2646
C_3	0.441	0.1764	0.1764	0.0882	0.3528	0.441
C_4	0.3528	0.3528	0.2646	0.3528	0.2646	0.2646
C_5	0.021	0.0105	-0.0105	0.0315	0.042	0.021
C_6	1.4112	3.528	2.646	1.0584	1.764	0.882
Min operator	5.6934 E-05	20.13502 E-5	-6.00476 E-06	9.60761 E-07	-2.56203 E-05	1.00079 E-5
Max operator	2.6138	4.2966	3.2172	1.6296	2.7657	1.8907
Values of the bipolar Choquet integral	2.61	4.30	3.22	1.63	2.77	1.89
Ranking of alternatives	4	1	2	6	3	5

In table 5 we show the comparisons between the ELECTRE IV and the bipolar Choquet rankings.

Table 5 - Comparison of results from ELECTRE IV against bipolar Choquet

Ranking from ELECTRE IV assuming that $C_1 > C_5 > C_6 > C_2 = C_3 = C_4$	Ranking from bipolar Choquet for H_1	Ranking from bipolar Choquet for H_2
Firm # 6	Firm # 2	Firm # 2
Firm # 3	Firm # 3	Firm # 3
Firms # 1 and 5	Firm # 5	Firm # 5
Firm # 4	Firm # 1	Firm # 1
Firm # 2	Firm # 6	Firm # 6
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4. Results

It can be seen that by using bipolar Choquet integral a complete pre-order is produced and ranking is different from that obtained by ELECTRE IV. In both hypotheses, when $C_1 > C_5 > C_6 > C_2 = C_3 = C_4$ (H_1) and when $C_6 > C_1 = C_5 > C_2 = C_3 = C_4$ (H_2), the bipolar Choquet integral leads to the same ranking. According to both approaches, Firm # 3 ranks as second alternative. Firm # 3 has low cost and low delay as compared against the other alternatives. Using ELECTRE IV leads to the conclusion that Firm # 6 ranks as first alternative. Firm # 6 has lowest cost and cost (C_1) is the most important criterion according to H_1. Considering time, the second most important criterion (C_5), Firm # 6 has the lowest value. Firm # 6 ranks in the fifth position by using bipolar Choquet. By using bipolar Choquet Firm # 2 is the best alternative, although its cost is three times higher than Firm # 6. On the other hand, Firm # 2 has four times more experience (C_6) than Firm # 6, and this is indeed the third most important criterion. Firm # 2 is also one and a half times lower in time (C_5), that is the second more important criterion; it also has more quality (C_4), that is the less important criterion. Firm # 2 has lower culture (C_2), that is the fourth important criterion, and design (C_3), that has relatively low importance as compared against other criteria. In the third position we have, from using ELECTRE IV, a tie: Firms # 1 and 5.

From using bipolar Choquet we obtain Firm # 5 in that third position. It would be fair to say that using Choquet bipolar is more realistic when we consider that Firm # 5 has lower price and higher experience. ELECTRE IV classifies Firm # 4 in the fourth position and Choquet bipolar classifies Firms # 1 in that same position. This firm has higher cost but has more experience, better time, design and culture. Finally, Firm # 2 is classified in the fifth position according to ELECTRE IV and bipolar Choquet classifies Firm # 2 in the first position as commented before.

5. Conclusions

We have shown through an application example that the uses of the bipolar Choquet integral and ELECTRE IV are comparable. The fact that interactions between criteria are taken into consideration when that first model is used leads to its advantage over ELECTRE IV. When the bipolar Choquet integral is used the tradeoffs between criteria are explicitly taken into account and measures of interactions associated to the fuzzy measures can be related related to two different hypotheses. For this particular application example we saw that interactions between criteria were indeed relevant. More research along the line pursued in this note should proceed along two major mainstreams: (i) carrying out similar and broader comparisons for larger problems; and (ii) focusing the comparison on multiple criteria decision aiding models that rely on bipolar Cumulative Prospect Theory [13].

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