Optimum Constant-stress Partially Accelerated Life Test for k Identical Repairable Systems

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Abstract

This paper considers optimal design for constant stress Partial Accelerated Life Test(PALT) on k identical repairable systems, wherein each system is run either at use or accelerated condition under failure truncated case. A 'minimal repairing' approach for the repairable system is assumed and Power Law Process (PLP) is modeled to describe the reliability change process of the repairable system. The optimum proportion of systems allocated to both normal use condition and accelerated condition for the constant PALT as well as optimum number of failures for each system is determined by using D-optimality criterion. The confidence intervals of the model parameters and the acceleration factor have also been obtained. The method developed has been illustrated using an example.

Keywords: partially accelerated life test, maximum likelihood estimation, fisher information matrix, asymptotic variance, non homogeneous poisson process, mean time between failure.

1. Introduction

Industrial competitiveness in terms of innovation, time of development, and field reliability expectations leads to high quality products. Thus, when a system is tested under normal operating conditions, then the amount of time for reliability assessment is usually quite large. In order to shorten life and/or accelerate performance degradation, the system under study is subjected to stresses which are more severe than usual like temperature, humidity, pressure, voltage, vibration etc. It is called accelerated life testing (ALT). The information obtained from the test performed in accelerated environment is used to predict actual product performance in usual environment. Nelson (1990) provides an extensive and comprehensive source for background material, practical methodology, basic theory, and examples for accelerated testing.

On the contrary, if we test products under accelerated conditions more test units are needed to obtain a sufficient number of failure data. In the designed and development phase, most of engineers have small prototype products. In this case, if we can repair the failed products and test those continuously, we can obtain appropriate failure data in short period. In many mechanical, electric, and electronic products, most failures at the system

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level are due to the failure of one component. For eg., an electronic board's failure induced by a capacitor failure can be easily repaired and testing can be resumed. A repairable system is a system that, after failing to perform one or more of its functions satisfactorily, can be restored to satisfactory performance even if the performance of system after repair may not be equal to that of the new one.

Several models of repair effect have been studied. Pham and Wang (1996) classified repair models as Perfect repair, Minimal repair, Imperfect repair, Worse repair, Worst repair. In reality, minimal repair (as bad as old) is of the most interest for repairable systems.

Under minimal repair, the failure process usually follows NHPP, such as Power Law Process (PLP). Only limited research has been found to address ALT for repairable systems. Among the limited research Guida and Giorgio (1995) analyzed the ALT data of a repairable item using Proportional Intensity model based on the PLP, under a single accelerating variable with two constant stress levels. Guerin et al (2004) studied ALT on repairable systems, defining two accelerated life models for repairable systems : the Arrhenius-exponential model and the Peck-Weibull model. Luo and Jiang (2009) studied the step stress accelerated life testing data analysis for the repairable systems. Balakrishnan et al (2009) introduced minimal repair under a simple step-stress test, based on exponential distributions and an associated cumulative exposure model.

However, there does not seem to exist any research paper in the literature which deals with partially accelerated life testing (PALT) on repairable systems. Unlike ALT where the test items are run only at accelerated condition, in a PALT systems are run at both use and accelerated conditions. The main aim of PALT is to collect more failure data in a limited time without necessarily using high stresses to all test units.

In this paper, optimum plan for constant-stress PALT on k identical repairable systems is discussed. We consider the sequence of failure times of a system under PALT in the sense of minimal repair, i.e. the system will instantaneously be repaired, and by this, put into the condition immediately prior to its failure. The times to repair are considered to be low and so are neglected. The PLP is modeled for the failure pattern of the repairable system. Although there are many environmental factors, we have used a single acceleration factor. This paper focuses on the maximum likelihood method for estimating the acceleration factor and parameters of the power law process with constant-stress PALT under failure truncated case. The optimum proportion of systems allocated to both normal use condition and accelerated condition for the constant PALT as well as optimum number of failures for each system is determined by using D-optimality criterion.

The rest of the paper is organized as follows: In Section 2 model's basic assumptions are described. In Section 3 model description is given. Section 4 presents the estimation method. In Section 5 variance-covariance matrix of the model parameters and acceleration factor is obtained. In Section 6 optimum test plans of simple constant-stress

PALT are developed. Confidence intervals of the model parameters and the acceleration factor are obtained in Section 7. Section 8 illustrates the method developed using the data simulated from the proposed models.

Notations

k	number of identical repairable systems put on test
n	total number of failures for each system
ϕ	proportion of systems allocated to use condition in a constant PALT
$1-\phi$	proportion of systems allocated to accelerated condition in a constant PALT
$k\phi$	number of systems allocated to use condition
$k(1-\phi)$	number of systems allocated to accelerated condition
A	Acceleration factor (A>1)
α , β	parameters of power law process
t_{ij}	j^{th} failure time of the i^{th} system
t_{in}	n^{th} failure time of the i^{th} system at which test is terminated
$F_u(t)$	cumulative distribution function at time t at use condition
$F_a(t)$	cumulative distribution function at time t at accelerated condition
$f_u(t)$	probability density function at time t at use condition
$f_a(t)$	probability density function at time t at accelerated condition
$\lambda_u(t)$	failure intensity at time t at use condition
$\lambda_a(t)$	failure intensity at time t at accelerated condition
t_a	failure time at accelerated condition
t_u	failure time at use condition
$m_u(t)$	expected number of failures at use condition at time t
$m_a(t)$	expected number of failures at accelerated condition at time t

2. Basic Assumptions

- All k repairable systems are identical and independent.
- The failures for system under test either at use condition or accelerated condition occur according to an NHPP with Power Law failure intensity.
- Failure time of a system at accelerated condition is $t_a = \frac{t_u}{A}$, $A > 1$.
- Each system is failure truncated at the n^{th} failure.

3. Model Description

The failures for system under study are occurring according to a NHPP with Power Law failure intensity

$$\lambda(t) = \left(\frac{\beta}{\theta}\right)\left(\frac{t}{\theta}\right)^{\beta-1}, \quad \text{with } \beta, \theta > 0. \tag{1}$$

This model is a useful model for phenomenon which is changing over time. Depending on the value of the shape parameter β, it can fit a wide range of failure processes. When $\beta < 1$, the failure intensity of the proposed model decreases with time in a continuous manner, fixed in advance, and describes the change in reliability over the entire development program irrespective of any fixes or design changes. When $\beta > 1$, the failure intensity increases with time and so the model can be used to the situations in which reliability deterioration is observed. Finally, when $\beta = 1$, the NHPP reduces to Homogeneous Poisson Process (HPP) with constant failure intensity.

The mean number of failures in time interval (0, t] are

$$m(t) = \int_0^t \lambda(u)du = \left(\frac{t}{\theta}\right)^{\beta}. \tag{2}$$

As each system is observed till n^{th} failure, let the n times to failure of i^{th} system be $t_{i1}, t_{i2}, t_{i3}, ..., t_{in}$ where $t_{i1} < t_{i2} < ... < t_{in}$ and i= 1, …, k.

For the i^{th} system, the joint probability density function of $t_{i1}, t_{i2}, t_{i3}, ..., t_{in}$ where $t_{i1} < t_{i2} < ... < t_{in}$ is

$$f(t_{i1}, t_{i2}, t_{i3}, ..., t_{in}) = \left[\prod_{j=1}^{n}\left(\frac{\beta}{\theta}\right)\left(\frac{t_{ij}}{\theta}\right)^{\beta-1}\right]\exp\left[-\left(\frac{t_{in}}{\theta}\right)^{\beta}\right]. \tag{3}$$

Therefore,

$$f(t_{ij}) = \int_{t_{i1}}\int_{t_{i2}}\int_{t_{i3}}....\int_{t_{i(j-1)}}\int_{t_{i(j+1)}}...\int_{t_{in}} f(t_{i1}, t_{i2}, t_{i3}, ..., t_{in})\, dt_{in}dt_{i(n-1)}...dt_{i(j+1)}dt_{i(j-1)}...dt_{i1}$$

$$= \frac{\beta t_{ij}^{j\beta-1}\exp\left[-\left(\frac{t_{ij}}{\theta}\right)^{\beta}\right]}{\theta^{j\beta}\Gamma(j)}, \text{ with } 0 < t_{ij} < \infty. \tag{4}$$

Since $t_a = \frac{t_u}{A}$, therefore

$$F_a(t_a) = F_u(At_a), \tag{5}$$

$$f_a(t_a) = Af_u(At_a), \tag{6}$$

$$\lambda_a(t_a) = A\,\lambda_u(At_a), \tag{7}$$

$$m_a(t_a) = m_u(At_a). \tag{8}$$

Thus, the failure intensity of the Power Law Process at time t_u at use condition (from (1)) is given by

$$\lambda_u(t_u) = \left(\frac{\beta}{\theta}\right)\left(\frac{t_u}{\theta}\right)^{\beta-1}, \quad \text{with } \beta, \theta > 0, \tag{9}$$

and at accelerated condition (from (1) & (7)) is

$$\lambda_a(t_a) = A\left(\frac{\beta}{\theta}\right)\left(\frac{At_a}{\theta}\right)^{\beta-1}, \quad \text{with } \beta, \theta > 0 \;\&\; A > 1. \tag{10}$$

For the i^{th} system, the density of a failure time at use condition (from (4)) is

$$f_u(t_{uij}) = \frac{\beta t_{uij}^{j\beta-1} \exp\left[-\left(\frac{t_{uij}}{\theta}\right)^{\beta}\right]}{\theta^{j\beta}\,\Gamma(j)}, \text{ with } 0 < t_{uij} < \infty, \tag{11}$$

and at accelerated condition (from (4) & (6)) is

$$f_a(t_{aij}) = \frac{A\beta (At_{aij})^{j\beta-1} \exp\left[-\left(\frac{At_{aij}}{\theta}\right)^{\beta}\right]}{\theta^{j\beta}\Gamma(j)}, \text{ with } 0 < t_{aij} < \infty. \tag{12}$$

Since $k\phi$ systems are allocated to use condition and $k(1-\phi)$ systems are allocated to accelerated condition, therefore, the likelihood function of the systems at use condition is

$$L_u = \prod_{i=1}^{k\phi}\left[\prod_{j=1}^{n}\lambda_u(t_{uij})\right]\exp[-m_u(t_{uin})]$$

$$= \prod_{i=1}^{k\phi}\left[\prod_{j=1}^{n}\left(\frac{\beta}{\theta}\right)\left(\frac{t_{uij}}{\theta}\right)^{\beta-1}\right]\exp\left[-\left(\frac{t_{uin}}{\theta}\right)^{\beta}\right], \tag{13}$$

and at accelerated condition is

$$L_a = \prod_{i=1}^{k(1-\phi)}\left[\prod_{j=1}^{n}\lambda_a(t_{aij})\right]\exp[-m_a(t_{ain})]$$

$$= \prod_{i=1}^{k(1-\phi)}\left[\prod_{j=1}^{n}A\left(\frac{\beta}{\theta}\right)\left(\frac{At_{aij}}{\theta}\right)^{\beta-1}\right]\exp\left[-\left(\frac{At_{ain}}{\theta}\right)^{\beta}\right]. \tag{14}$$

Since the systems are independent, the likelihood function for k systems is given by

$$L = L_u.L_a$$

$$= \left[\prod_{i=1}^{k\phi}\left[\prod_{j=1}^{n}\left(\frac{\beta}{\theta}\right)\left(\frac{t_{uij}}{\theta}\right)^{\beta-1}\right]\exp\left[-\left(\frac{t_{uin}}{\theta}\right)^{\beta}\right]\right]$$

$$.\left[\prod_{i=1}^{k(1-\phi)}\left[\prod_{j=1}^{n}A\left(\frac{\beta}{\theta}\right)\left(\frac{At_{aij}}{\theta}\right)^{\beta-1}\right]\exp\left[-\left(\frac{At_{ain}}{\theta}\right)^{\beta}\right]\right]. \tag{15}$$

4. Estimation Method

Maximum Likelihood method has been used to estimate the model parameters, β, θ, and acceleration factor, A.

It is usually easier to maximize the natural logarithm of the likelihood function rather than the likelihood function itself. The log likelihood function of (15) is

$$\log(L)=\sum_{i=1}^{k\phi}\left(n\log\beta-n\beta\log\theta-\left(\frac{t_{uin}}{\theta}\right)^{\beta}+(\beta-1)\sum_{j=1}^{n}\log t_{uij}\right)$$
$$+\sum_{i=1}^{k(1-\phi)}\left(n\log\beta-n\beta\log\theta+n\beta\log A-\left(\frac{At_{ain}}{\theta}\right)^{\beta}+(\beta-1)\sum_{j=1}^{n}\log t_{aij}\right). \qquad (16)$$

The maximum likelihood equations are

$$\frac{\partial\log(L)}{\partial\beta}=\sum_{i=1}^{k\phi}\left(\frac{n}{\beta}-n\log\theta-\left(\frac{t_{uin}}{\theta}\right)^{\beta}\log\left(\frac{t_{uin}}{\theta}\right)+\sum_{j=1}^{n}\log t_{uij}\right)$$
$$+\sum_{i=1}^{k(1-\phi)}\left(\frac{n}{\beta}-n\log\theta+n\log A-\left(\frac{At_{ain}}{\theta}\right)^{\beta}\log\left(\frac{At_{ain}}{\theta}\right)+\sum_{j=1}^{n}\log t_{aij}\right)$$
$$=0. \qquad (17)$$

$$\frac{\partial\log(L)}{\partial\theta}=\sum_{i=1}^{k\phi}\left(\frac{-n\beta}{\theta}+\beta\left(\frac{t_{uin}}{\theta^2}\right)\left(\frac{t_{uin}}{\theta}\right)^{\beta-1}\right)$$
$$+\sum_{i=1}^{k(1-\phi)}\left(\frac{-n\beta}{\theta}+A\beta\left(\frac{t_{ain}}{\theta^2}\right)\left(\frac{At_{ain}}{\theta}\right)^{\beta-1}\right)$$
$$=0. \qquad (18)$$

$$\frac{\partial\log(L)}{\partial A}=\sum_{i=1}^{k(1-\phi)}\left(\frac{n\beta}{A}-\beta\left(\frac{t_{ain}}{\theta}\right)\left(\frac{At_{ain}}{\theta}\right)^{\beta-1}\right)=0. \qquad (19)$$

To find out the maximum likelihood estimates $\hat{\beta}, \hat{\theta}, \hat{A}$, we have to solve the above system of nonlinear equations (17 - 19) with respect to β, θ, and A. As this system has no closed form solution in β, θ, and A, therefore the maximum likelihood estimates $\hat{\beta}, \hat{\theta}, \hat{A}$, are obtained by maximizing (16) for which Maximize option of *Mathematica 6.0* has been used with appropriate restrictions on parameters.

5. Fisher Information Matrix

It is the 3×3 symmetric matrix of expectation of negative second order partial derivatives of the log likelihood function with respect to β, θ, and A.

$$F(\beta,\theta,A)=\begin{pmatrix} E\left[-\frac{\partial^2 \log(L)}{\partial\beta^2}\right] & E\left[-\frac{\partial^2 \log(L)}{\partial\beta\partial\theta}\right] & E\left[-\frac{\partial^2 \log(L)}{\partial\beta\partial A}\right] \\ E\left[-\frac{\partial^2 \log(L)}{\partial\theta\partial\beta}\right] & E\left[-\frac{\partial^2 \log(L)}{\partial\theta^2}\right] & E\left[-\frac{\partial^2 \log(L)}{\partial\theta\partial A}\right] \\ E\left[-\frac{\partial^2 \log(L)}{\partial A\partial\beta}\right] & E\left[-\frac{\partial^2 \log(L)}{\partial A\partial\theta}\right] & E\left[-\frac{\partial^2 \log(L)}{\partial A^2}\right] \end{pmatrix}, \quad (20)$$

where

$$E\left[-\frac{\partial^2 \log(L)}{\partial\beta^2}\right]=\frac{nk\phi}{\beta^2}+\frac{nk(1-\phi)}{\beta^2}$$
$$+\left[\sum_{i=1}^{k\phi}\frac{\beta}{\theta^{(n+1)\beta}\Gamma(n)}\int_0^\infty t_{uin}^{(n+1)\beta-1}\left[\log\left(\frac{t_{uin}}{\theta}\right)\right]^2\exp\left[-\left(\frac{t_{uin}}{\theta}\right)^\beta\right]dt_{uin}\right]$$
$$+\left[\sum_{i=1}^{k(1-\phi)}\frac{\beta A^{(n+1)\beta}}{\theta^{(n+1)\beta}\Gamma(n)}\int_0^\infty t_{ain}^{(n+1)\beta-1}\left[\log\left(\frac{At_{ain}}{\theta}\right)\right]^2\exp\left[-\left(\frac{At_{ain}}{\theta}\right)^\beta\right]dt_{ain}\right]$$
$$=\frac{nk}{\beta^2}\left[1+\left[\text{PolyGamma}[0,n+1]\right]^2+\text{PolyGamma}[1,n+1]\right],$$

$$E\left[-\frac{\partial^2 \log(L)}{\partial\beta\partial\theta}\right]=-\left[\sum_{i=1}^{k\phi}\frac{\beta^2}{\theta^{(n+1)\beta+1}\Gamma(n)}\int_0^\infty t_{uin}^{(n+1)\beta-1}\log\left(\frac{t_{uin}}{\theta}\right)\exp\left[-\left(\frac{t_{uin}}{\theta}\right)^\beta\right]dt_{uin}\right]$$
$$-\left[\sum_{i=1}^{k(1-\phi)}\frac{\beta^2 A^{(n+1)\beta}}{\theta^{(n+1)\beta+1}\Gamma(n)}\int_0^\infty t_{ain}^{(n+1)\beta-1}\log\left(\frac{At_{ain}}{\theta}\right)\exp\left[-\left(\frac{At_{ain}}{\theta}\right)^\beta\right]dt_{ain}\right]$$
$$=\frac{-nk}{\theta}\text{PolyGamma}[0,n+1],$$

$$E\left[-\frac{\partial^2 \log(L)}{\partial\beta\,\partial A}\right]=\left[\sum_{i=1}^{k(1-\phi)}\frac{\beta^2 A^{(n+1)\beta-1}}{\theta^{(n+1)\beta}\Gamma(n)}\int_0^\infty t_{ain}^{(n+1)\beta-1}\log\left(\frac{At_{ain}}{\theta}\right)\exp\left[-\left(\frac{At_{ain}}{\theta}\right)^\beta\right]dt_{ain}\right]$$

$$=\frac{nk(1-\phi)}{A}\text{PolyGamma}[0, n+1],$$

$$E\left[-\frac{\partial^2 \log(L)}{\partial\theta\partial\beta}\right]=E\left[-\frac{\partial^2 \log(L)}{\partial\beta\partial\theta}\right],$$

$$E\left[-\frac{\partial^2 \log(L)}{\partial\theta^2}\right]=\frac{nk\beta^2}{\theta^2},$$

$$E\left[-\frac{\partial^2 \log(L)}{\partial\theta\partial A}\right]=\frac{-nk(1-\phi)\beta^2}{A\theta},$$

$$E\left[-\frac{\partial^2 \log(L)}{\partial A\partial\beta}\right]=E\left[-\frac{\partial^2 \log(L)}{\partial\beta\partial A}\right],$$

$$E\left[-\frac{\partial^2 \log(L)}{\partial A\partial\theta}\right]=E\left[-\frac{\partial^2 \log(L)}{\partial\theta\partial A}\right],$$

$$E\left[-\frac{\partial^2 \log(L)}{\partial A^2}\right]=\frac{nk(1-\phi)\beta^2}{A^2}.$$

In *Mathematica 6.0*, the function PolyGamma[z] gives the digamma function

$$\psi(z)=\frac{d}{dz}\big(\log\Gamma(z)\big)=\frac{\Gamma'(z)}{\Gamma(z)}. \tag{21}$$

PolyGamma[n, z] gives the n^{th} derivative of the digamma function $\psi^n(z)$.

6. Optimum Plan

The optimum test plan can be obtained by using variance optimality criterion, D-optimality criterion or A-optimality criterion. These optimal plans are considered as minimizing or maximizing the selected objective functions that are purely based on Fisher information matrix. In this paper, D-optimality criterion has been used. The optimum 'ϕ' is found by minimizing the reciprocal of the determinant of the Fisher Information Matrix of the model parameters, i.e.,

$$\text{Min. } \frac{1}{|F|}$$

$$\text{s.t.} \qquad \begin{aligned} & 0 < \phi < 1 \\ & k\phi \in z^{+} \end{aligned} \tag{22}$$

For this, we have used Minimize option of *Mathematica 6.0*.

7. Confidence Intervals

The MLEs β, θ , and A are approximately normally distributed in large samples, therefore $(\hat{\beta}, \hat{\theta}, \hat{A}) \sim N((\beta, \theta, A), F^{-1})$. The two sided 100(1- α) % approximate confidence interval for the parameter β is given by $\hat{\beta} \pm z_{\alpha/2}\sqrt{\hat{Var}(\hat{\beta})}$, where $z_{\alpha/2}$ is the $(\alpha/2)^{th}$ quantile of a standard normal distribution, and $\sqrt{\hat{Var}(\hat{\beta})}$ is obtained by taking square root of the first diagonal element of inverse of estimated Fisher information matrix, $\hat{F}^{-1}$. Similarly two-sided 100(1 − α) % approximate confidence interval for the parameter θ and acceleration factor A can be obtained.

Although this method is quick and easy, one major problem associated with it is that it does not necessarily take the parameter space into account when constructing confidence intervals. There is no built-in procedure to prevent this and as a result, the lower bounds of the approximate confidence intervals frequently hit below zero though the parameter can take only positive values. In order to turn such intervals into sensible ones, the negative lower bounds are replaced by zero. In case of acceleration factor A, lower bound less than 1 is replaced by 1.

However, Meeker and Escobar (1998) have suggested the use of a log transformation to obtain approximate confidence intervals for the parameters that take positive values. Thus, the approximate two sided 100(1 - α) % confidence intervals for β is

$$\left[\hat{\beta}\exp\left[-z_{\alpha/2}\frac{\sqrt{\hat{Var}(\hat{\beta})}}{\hat{\beta}}\right], \hat{\beta}\exp\left[z_{\alpha/2}\frac{\sqrt{\hat{Var}(\hat{\beta})}}{\hat{\beta}}\right]\right]. \tag{23}$$

Similarly two-sided $100(1-\alpha)$ % approximate confidence interval for the parameter θ and acceleration factor A can be obtained. In case of acceleration factor A, lower bound less than 1 is replaced by 1.

8. Numerical Example

The method developed has been illustrated using the following data set:

$$n = 35, k = 6, \beta = 0.6, \theta = 0.2, A = 3.$$

8.1. Optimum Plan

The optimum value of 'ϕ' came out to be 0.5 respectively.

Therefore, $k\phi = 3$ and $k(1-\phi) = 3$, i.e., 3 systems are allocated at use condition and 3 systems at accelerated condition.

8.2. Maximum Likelihood Estimation

Based on n = 35, $\beta = 0.6, \theta = 0.2, A = 3$, the failure times for systems at use condition and accelerated condition are simulated. The data in Table 1 includes 35 simulated failure times for 6 systems. The first 3 systems are allocated at use condition and the rest 3 at accelerated condition.

Table 1: Simulated failure times for $(n = 35, k = 6, \beta = 0.6, \theta = 0.2, A = 3, \phi = 0.5)$

Constant-stress		Failure times						
Use condition	System 1	1.57,	4.99,	5.54,	5.95,	6.67,	6.77,	7.39,
		7.50,	12.26,	12.88,	14.42,	15.10,	23.30,	23.97,
		24.47,	27.17,	27.24,	27.32,	27.39,	27.94,	28.11,
		29.05,	30.52,	40.19,	40.79,	42.02,	42.28,	45.74,
		50.17,	51.20,	55.22,	58.37,	64.06,	79.50,	80.09
	System 2	0.05,	0.79,	1.68,	4.73,	5.97,	8.32,	10.15,
		14.60,	15.66,	18.39,	19.35,	20.70,	22.01,	23.82,
		23.86,	24.97,	28.29,	32.08,	35.50,	38.41,	38.79,
		48.01,	52.15,	54.38,	59.65,	60.06,	60.11,	62.79,
		63.43,	64.64,	65.94,	69.38,	74.18,	77.06,	87.99
	System 3	0.03,	0.10,	4.38,	7.82,	10.06,	11.32,	11.72,
		12.10,	12.30,	13.22,	13.74,	14.85,	16.59,	18.42,
		18.71,	19.90,	19.99,	21.17,	25.15,	25.44,	26.66,
		29.41,	30.22,	30.67,	31.39,	32.64,	33.50,	34.21,
		35.92,	41.18,	41.38,	41.50,	42.80,	44.50,	49.49

Accelerated condition	**System 1**	0.03,	0.23,	0.67,	1.01,	1.07,	1.55,	1.77,
		2.60,	2.62,	2.81,	3.52,	3.98,	5.21,	5.26,
		5.54,	5.76,	6.43,	7.07,	7.57,	7.75,	8.42,
		8.62,	8.66,	9.07,	14.54,	15.13,	15.35,	16.62,
		18.54,	19.94,	21.62,	24.01,	25.96,	26.59,	26.65
	System 2	0.13,	0.74,	0.86,	1.26,	1.27,	1.40,	1.94,
		2.04,	2.81,	3.46,	3.86,	3.94,	4.25,	5.01,
		5.05,	6.88,	7.22,	9.31,	9.64,	9.78,	11.17,
		11.23,	13.55,	13.66,	14.31,	14.45,	15.31,	15.85,
		16.49,	18.31,	20.32,	20.61,	21.83,	21.97,	22.42
	System 3	0.01,	0.03,	0.09,	0.13,	0.16,	0.32,	0.49,
		1.33,	1.35,	1.92,	2.18,	2.19,	3.29,	3.45,
		3.67,	3.98,	4.35,	5.31,	5.75,	6.33,	6.52,
		6.61,	6.95,	7.15,	7.26,	8.77,	12.07,	15.77,
		20.06,	20.44,	21.11,	21.16,	25.56,	26.45,	39.93

The MLEs of model parameters β, θ , and acceleration factor A obtained using Maximize option of *Mathematica 6.0* are

$$\hat{\beta} = 0.621818,\ \hat{\theta} = 0.235868,\ \hat{A} = 2.44609\,.$$

8.3. Estimated Expected Number of Failures at Use Condition v/s Accelerated Condition

Estimated expected number of failure at use condition = $\left(\frac{t_u}{\hat{\theta}}\right)^{\hat{\beta}}$.

Estimated expected number of failure at accelerated condition = $\left(\frac{At_a}{\hat{\theta}}\right)^{\hat{\beta}}$.

Figure 1 clearly shows that it takes 'A = 2.44' times longer to obtain same estimated expected number of failures at use condition than at accelerated condition.

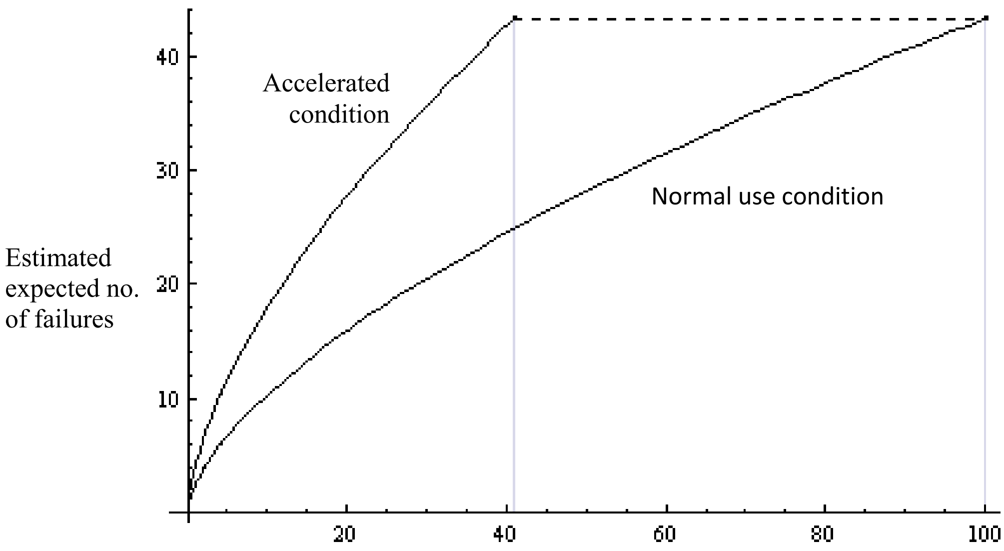

Figure 1: Estimated expected number of failures at use condition and accelerated condition

8.4. Confidence Intervals

Since MLEs are asymptotically normally distributed, therefore approximate confidence intervals can be used to find confidence intervals for the parameters β, θ , and acceleration factor A, as sample size is large.

The observed values of F^{-1}, i.e. $\hat{F}^{-1}$, were determined by substituting the estimated parameters and estimated acceleration factor $\hat{\beta}, \hat{\theta}, \hat{A}$, for the true parameters in the asymptotic covariance matrix.

$$\hat{F}^{-1} = \begin{pmatrix} 0.00179078 & 0.00389942 & 2.55658 \text{ x } 10^{-17} \\ 0.00389942 & 0.0098613 & 0.014211 \\ 3.22936 \text{ x } 10^{-17} & 0.014211 & 0.294754 \end{pmatrix}.$$

The two sided 95% confidence intervals using Normal approximation and Log Normal approximation are given in Table 2.

Table 2: Two sided 95 % confidence intervals

Parameter	Actual values of parameters	Confidence intervals			
		Using normal approximation		Using log normal approximation	
		Lower limit	Upper limit	Lower limit	Upper limit
β	0.6	0.538875	0.70476	0.5441689	0.710547
θ	0.2	0.0412316	0.430504	0.103346	0.538325
A	3	1.38198	3.5102	1.58323	3.77921

9. Conclusion

In this paper we have formulated the optimal design of constant-stress PALT for k identical repairable systems using PLP model and with failure truncated data set. It has been found through a numerical example that it takes much longer to obtain same estimated expected number of failures at use condition than at accelerated condition.

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